Loops

Zygmunt Morawski

ABSTRACT: One has solved the Dirac equation in the case of loops in the cylindrical coordinates using the method of the separation of the variables. One has postulated a shape of wave function and has solved the Dirac equation in consideration of potential. Next, one has stated that in the geometrical center of the loop there exists the singularity, which testifies to the existence of a particle connected with the loop. Moreover, one has proved the existence of the confinement of loops, which has permitted to describe the confinement of ghosts.

The Dirac equation has the shape:

$$\left(\vec{\alpha} \cdot \vec{\nabla} + \beta m + V\right) \Psi = E\Psi \qquad (1)$$

We consider the cylindrical coordinates ϱ , φ , z [1] :

$$\nabla = e_1 \frac{\partial \Psi}{\partial \varrho} + e_2 \frac{1}{\varrho} \frac{\partial \Psi}{\partial \varphi} + e_3 \frac{\partial \Psi}{\partial z} \qquad (2)$$

The origin of the coordinate system is placed in the geometrical center of the loop (Fig.1). Moreover, we assume that:

$$\Psi = \Psi_1 = \Psi_2 = \Psi_3 = \Psi_4$$

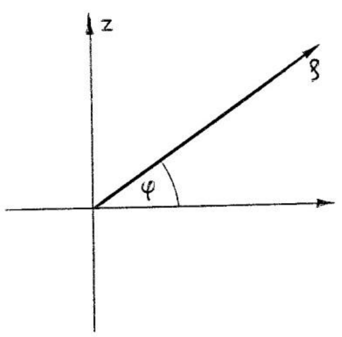

Fig.1

We will solve the equation with the method of the separation of variables.

So we have:

$$\psi = \psi_1(\varrho) \ \psi_2(\varphi) \ \psi_3(z)$$

$$V = V_1(\varrho) + V_2(\varphi) + V_3(z)$$

We put m = 0, because the loop can't have the nonzero rest mass, because it needs to move with the velocity v = c in order not to be a distinguished reference system [2].

Our considerations are valid for each loop not necessarily moving with the velocity

v = c, because if m ≠ 0, the energy E is only rescaled.

So we have:

$$(\nabla + V)\Psi = \frac{\partial \Psi}{\partial t} \qquad (5)$$

and

$$\Psi = \Psi_s \cdot \Psi_t = e^{\frac{-iEt}{\hbar}} \psi_1(\varrho)\ \psi_2(\varphi)\ \psi_3(z) \qquad (6)$$

The equation (5) obtains the shape:

$$\frac{\partial \Psi}{\partial \varrho} + \frac{1}{\varrho}\frac{\partial \Psi}{\partial \varphi} + \frac{\partial \Psi}{\partial z} + V\Psi = \frac{\partial \Psi}{\partial t} \qquad (7)$$

We put (6) into (7) and obtain:

$$\frac{\partial \psi_1(\varrho)}{\partial \varrho} e^{\frac{-iEt}{\hbar}} \psi_2(\varphi)\ \psi_3(z) + \frac{1}{\varrho}\frac{\partial \psi_2(\varphi)}{\partial \varphi} \psi_1(\varrho)\ \psi_3(z)\ e^{\frac{-iEt}{\hbar}}$$

$$+ \frac{\partial \psi_3(z)}{\partial z} e^{\frac{-iEt}{\hbar}} \psi_1(\varphi)\ \psi_2(\varphi) = -\frac{iE}{\hbar} e^{\frac{-iEt}{\hbar}} \psi_1(\varrho)\ \psi_2(\varphi)\ \psi_3(z) \qquad (8)$$

We divide both members of the equation (8) by $\Psi(\varrho, \varphi, z, t)$ expressed by the formula (6). We obtain:

$$\frac{1}{\psi_1(\varrho)}\frac{\partial \psi_1(\varrho)}{\partial \varrho} + \frac{1}{\varrho}\frac{1}{\psi_2(\varphi)}\frac{\partial \psi_2(\varphi)}{\partial \varphi} + \frac{1}{\psi_3(z)}\frac{\partial \psi_3(z)}{\partial z} + V = \frac{E}{\hbar} \qquad (9)$$

and we put:

$$\frac{E}{\hbar} = E' = E_\varrho + E_\varphi + E_z \tag{10}$$

Using consequently the next separation we obtain three equations:

$$\frac{1}{\psi_1(\varrho)} \frac{\partial\psi_1(\varrho)}{\partial\varrho} + V_\varrho = E_\varrho$$

$$\frac{1}{\varrho} \frac{1}{\psi_2(\varphi)} \frac{\partial\psi_2(\varphi)}{\partial\varphi} + V_\varphi = E_\varphi$$

$$\frac{1}{\psi_3(z)} \frac{\partial\psi_3(z)}{\partial z} + V_z = E_z$$

So:

$$\frac{\partial\psi_1(\varrho)}{\partial\varrho} + V_\varrho \psi_1(\varrho) = E_\varrho\psi_1(\varrho) \tag{11}$$

$$\frac{1}{\varrho} \frac{\partial\psi_2(\varphi)}{\partial\varphi} + V_\varphi \psi_2(\varphi) = E_\varrho\psi_2(\varphi) \tag{12}$$

$$\frac{\partial\psi_3(z)}{\partial z} + V_z \psi_3(z) = E_z\psi_3(z) \tag{13}$$

The idea consists in the assumption that we know the wave function and we want to find the potential, so differently than usually.
We postulate:

$$\psi_1(\varrho) = \alpha\varrho \tag{14}$$

because the periphery of the loop is equal $2\pi\varrho = \alpha\varrho$.
We put (14) into (11) and obtain:

$$\alpha + V_{1\varrho}\, \alpha\varrho = E_\varrho\, \alpha\varrho$$

so:

$$V_\varrho = E_\varrho - \frac{1}{\varrho} \tag{15}$$

We have the singularity for $\varrho = 0$, what testifies to that a particle is connected with the loop. So we have the dualism loop-particle. In the geometrical center of the loop there is a crater of potential.

Next, we put $\psi_2(\varphi)$ = const, what is implicated by the symmetry of the problem, into (12). We obtain:

$$V_2\ (\varphi) = E_\varphi = \text{const}$$

If we take $\psi_3(z) = e^{kz} + e^{-kz}$ or $\psi_3(z) = e^{ikz} + e^{-ikz}$ and put them into (13), we obtain:

$$V_3\ (z) = \text{const}$$

Another solution is:

$$\psi_3(z) = \beta_z \tag{16}$$

From (13) and (16) we obtain:

$$V_3(z) = E_z - \frac{1}{z} \tag{17}$$

Yet another solution:

$$\psi_3(z) = \frac{\gamma}{z}$$

$$V_3(z) = E_z + \frac{1}{z} \tag{18}$$

The formulas (15), (17) and (18) support the idea that we have confined particles in the infinite potential of the selfinteraction. So we have the loops of ghosts and the confined ghosts.

At the price of the assumption that loops move with the velocity v=c we have obtained the description of the loops of ghosts, about which it isn't known if they move with the limit velocity. But it doesn't change anything.

If $m \neq 0$ and m=const, mass in the equation (1) causes only the change of the scale of energy, which is here only a parameter.

We have assumed the oscillating character of the loops in the function of time.

Acknowledgements:

Herewith I have published all mathematical details of my considerations, because I am ashamed of the fellow workers theorists who hide in their publications the details of their calculations.

References:

[1] J. D. Björken, S. D. Drell, “Relativistic Quantum Mechanics“
[2] A. K. Wróblewski, J. A. Zakrzewski, „Wstęp do fizyki”, t. 1